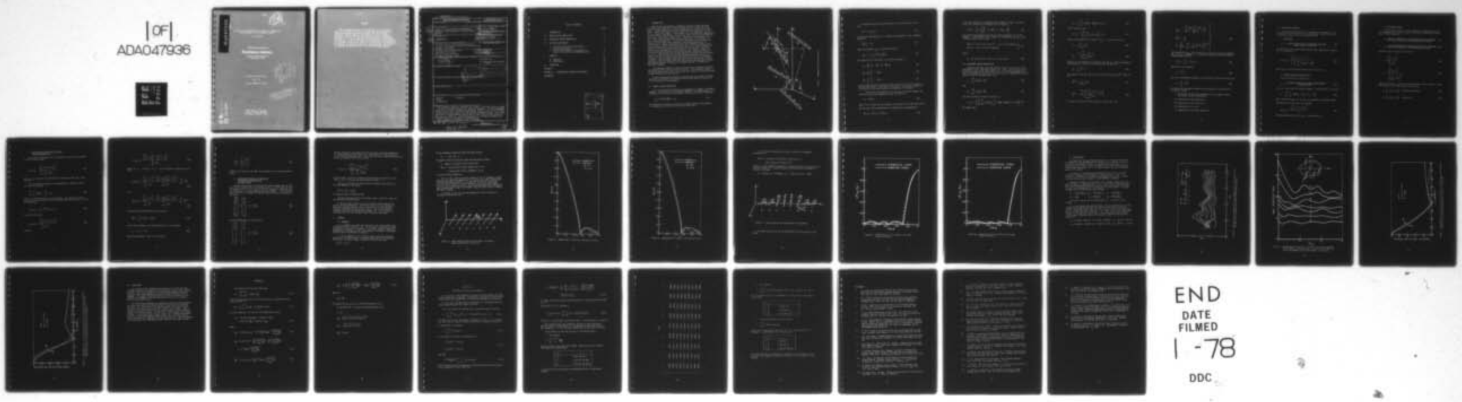
AD-A047 936 OHIO STATE UNIV COLUMBUS ELECTROSCIENCE LAB F/G 9/5
SYNTHESIS AND OPTIMIZATION FOR ARRAYS OF NONPARALLEL WIRE ANTEN--ETC(U)
JUN 77 J SAHALOS N00014-76-C-0573
UNCLASSIFIED ESL-4372-3 NL

1 OF 1
ADA047936



END
DATE
FILMED
1-78
DDC

ADA047936

(12)

SYNTHESIS AND OPTIMIZATION FOR ARRAYS OF NONPARALLEL WIRE ANTENNAS BY THE ORTHOGONAL METHOD

John Sahalos

The Ohio State University

ElectroScience Laboratory

Department of Electrical Engineering
Columbus, Ohio 43212

Technical Report 4372-3

June 1977

Contract N00014-76-C-0573

Department of the Army
Office of Naval Research
Arlington, Virginia 22217

NOTICES

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 SYNTHESIS AND OPTIMIZATION FOR ARRAYS OF NONPARALLEL WIRE ANTENNAS BY THE ORTHOGONAL METHOD.		5. TYPE OF REPORT & PERIOD COVERED 9 Technical ~~Report~~,
		6. PERFORMING ORG. REPORT NUMBER 14 ESL-4372-3
7. AUTHOR(s) 10 John Sahalos		8. CONTRACT OR GRANT NUMBER(s) 15 N00014-76-C-0573
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering Columbus, Ohio 43212		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Department of the Army Office of Naval Research Arlington, Virginia 22217		12. REPORT DATE 11 June 1977
		13. NUMBER OF PAGES 33
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 12 37p.		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

DISTRIBUTION STATEMENT A

Approved for public release; Distribution Unlimited

DDC

DEC 22 1977

F

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Antennas
Arrays
Optimization
Synthesis

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The orthogonal method is presented for synthesis and optimization problems of arrays of straight, nonparallel, thin-wire antennas. First by the moment method the far zone field and the performance indices of the array are represented by expressions useful for applying the orthogonal method. By a procedure similar to Gram-Schmidt method, the far zone field and the indices of the array are modified. The new expressions by a suitable procedure give helpful formulas for synthesis and optimization.

Several examples for possible uses are included.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

402 251

LB

TABLE OF CONTENTS

I. INTRODUCTION

Optimization and synthesis methods for antenna arrays have been studied by many researchers; the existance alone of a long series of papers on these subjects is enough to emphasize the importance of these areas. Specifically Dolph [1] and Riblet [2] have published a series of very interesting papers. These papers, by using Chebyshev polynomials, give a synthesis for uniform linear arrays which offers the minimum beam-width for a prescribed sidelobe level. Unz[3-6], Harrington[7], King, et al. [8], Jacabsen and Madsey [9] and many others gave answers to very interesting problems. On the array optimization Uzkov [10] first found by using linear transformations, the maximum directivity of linear arrays. Uzkov's theoretical work was extended by Bloch, et al. [11], Uzsok and Solymar [12] and Stearns [13]. Tai [14] considered the problem of achieving maximum directivity in uniform linear arrays of short dipoles and gave many useful graphs on the subject. Cheng [15] studied the determination of directivity in more complicated arrays. Lo, et al. [16] obtained the optimum SNR with constraint on the Q-factor. Sungiri and Butler [17] have used the eigenvalue method to find the solution for the maximum directivity with constraints on the resulting sidelobes. Matrix methods were applied by Strait and Kuo [18], Sarkar and Strait [19] and Sahalos [20] for constrained optimization of various performance indices of arrays with straight parallel or nonparallel thin wire antennas.

The orthogonal method for arrays consisting of arbitrarily oriented short dipoles was applied by Sahalos [21-22]. The orthogonal method one decade ago was used first by Unz [23] and recently by Sahalos [24-26] in many antenna array applications.

In the present work an effort is made to give some useful formulas applicable to synthesis and optimization problems when the arrays consist of nonparallel wire antennas.

II. MOMENT METHOD FORMULATION

Let a wire structure (Figure 1) be composed of a number of straight segments. By defining a right handed orthogonal coordinate system $(\hat{n},\hat{\phi},\hat{\ell})$ at each point of the wire's cylindrical surface, we can find [27] that:

$$-\int_0^L I(\ell)(E_\ell^m - Z_S H_\phi^m)\,d\ell = V_m \qquad . \qquad (1)$$

This expression expresses the reaction integral equation developed by J. Richmond and is true for electrically thin wires.

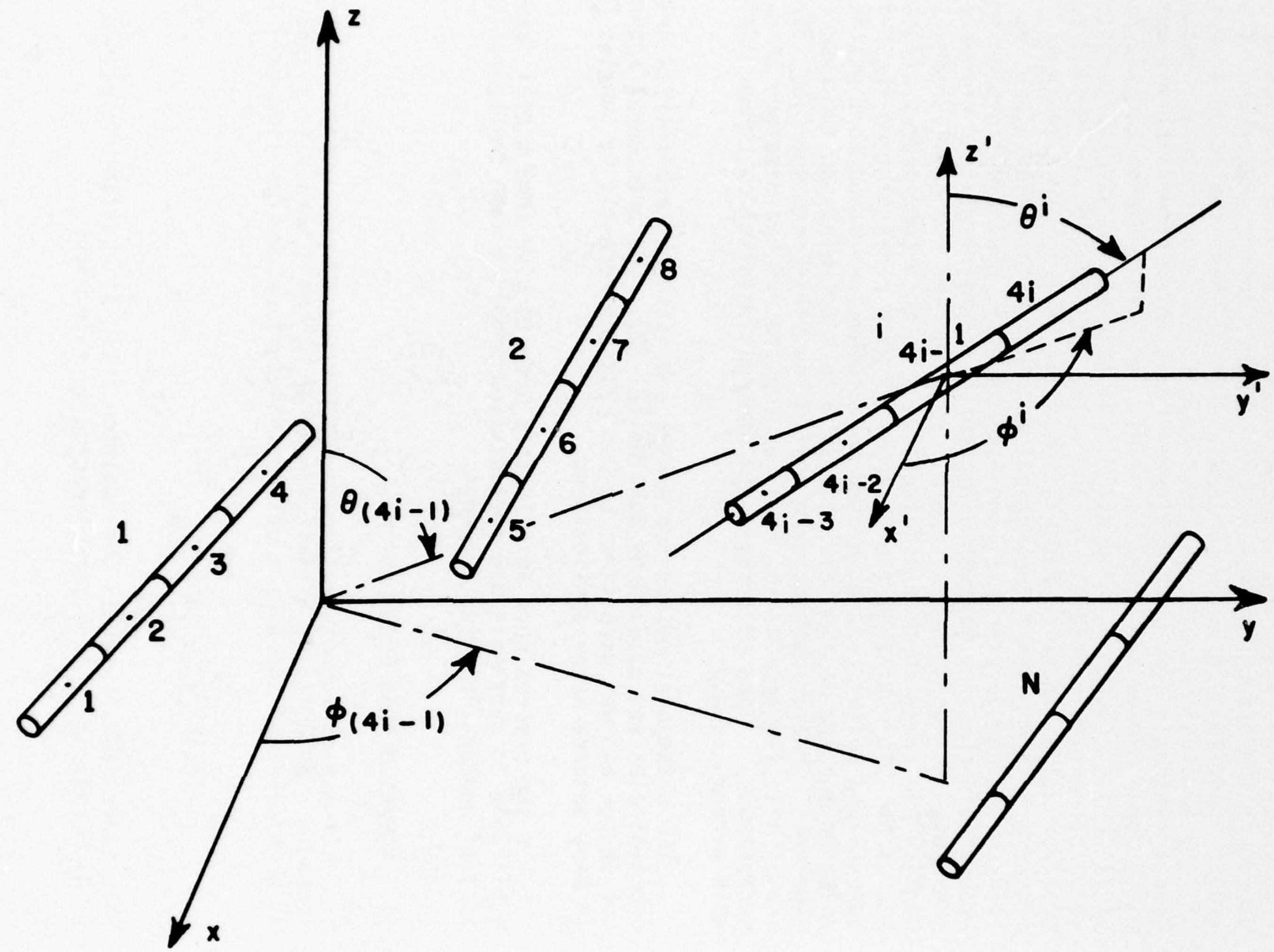


Figure 1. Geometry of a wire structure.

By expanding the current on the wire in a finite series of the form

$$I(\ell) = \sum_n I_n F_n(\ell) \tag{2}$$

we can have from Equation (1) a system of simultaneous linear algebraic equations

$$[Z](I) = (V) \tag{3}$$

where the elements Z_{mn} of the matrix $[Z]$ are:

$$Z_{mn} = - \int_n F_n(\ell)(E^m - Z_S H_\phi^m) d\ell \quad . \tag{4}$$

The quantities in Equations (1) and (4) are given by:

$$V_m = \iiint (\vec{J}_i \cdot \vec{E}_i^m - \vec{M}_i \cdot \vec{H}_i^m) \, dv \tag{5}$$

$$E_m^\ell = \frac{1}{2\pi} \int_0^{2\pi} \hat{\ell} \cdot \vec{E}^m \, d\phi \tag{6}$$

$$E_m^\phi = \frac{1}{2\pi} \int_0^{2\pi} \hat{\phi} \cdot \vec{H}^m \, d\phi \tag{7}$$

where $(\vec{J}_i, \vec{M}_i)$ are the impressed currents by which the wire structure is excited and $(E^m, \vec{H}^m)$ is the field of an electric test source located in the interior of the wire surface and radiating in free space.

By the help of Equation (3) we can find a relation between the currents and the corresponding voltages of the form:

$$(I) = [Y](V) \tag{8}$$

where in the column (V) the elements are nonzero only at the feed points.

The far zone field transmitted by an antenna can be expressed as:

$$\vec{E}(\phi,\theta) = \vec{E}^\theta(\phi,\theta) + \vec{E}^\phi(\phi,\theta) \quad . \tag{9}$$

If the wire structure is composed of N1 segments and N(N < N1) main ports, the expression of Equation (9) will become:

$$\vec{E}(\phi,\theta) = \sum_{i=1}^{N} V_i \left\{ \sum_{n=1}^{N1} Y_{ni} \, [\vec{E}_n^{\theta}(\phi,\theta) + \vec{E}_n^{\phi}(\phi,\theta)] \right\} \quad . \tag{10}$$

The vectors $\vec{E}_n^{\theta}$ and $\vec{E}_n^{\phi}$ are the electric field components of the n^{th} segment. If the center of the segment has coordinates (r_n, ϕ_n, θ_n) and direction angles (ϕ^n, θ^n) then:

$$\left.\begin{aligned} \vec{E}_n^{\theta}(\phi,\theta) &= [\cos\theta \, \sin\theta^i \, \cos(\phi-\phi^i) - \sin\theta \, \cos\theta^i] \exp[jq_i]\hat{\theta} \\ \vec{E}_n^{\phi}(\phi,\theta) &= \sin\theta^i \, \sin(\phi^i-\phi) \exp[jq_i]\hat{\phi} \end{aligned}\right\} \tag{11}$$

where:

$$q_i = \frac{2\pi}{\lambda} r_i[\sin\theta \, \sin\theta_i \, \cos(\phi-\phi_i) + \cos\theta \, \cos\theta_i] \quad . \tag{12}$$

III. ORTHOGONAL METHOD FORMULATION

Equation (10) shows that the electric field is a vector of an N1-dimensional vector space with a basis $\{\vec{O}_i\} = \{\vec{E}_{i\theta}^{\theta} + \vec{E}_i^{\phi}\}$. As we can see *N vectors of this space define the $\vec{E}(\phi,\theta)$. By a method analogous to* that of orthogonality we can express the field as a situation of N orthogonal vectors of the same space. Let

$$\vec{\phi}_i = \sum_{n=1}^{N1} Y_{ni}(\vec{E}_n^{\theta} + \vec{E}_n^{\phi}) \tag{13}$$

and

$$\vec{\phi}_j = \sum_{m=1}^{N1} Y_{mj}(\vec{E}_m^{\theta} + \vec{E}_m^{\phi}) \quad . \tag{14}$$

The inner product of these two vectors is:

$$\langle\vec{\phi}_i, \vec{\phi}_j\rangle = \left[\sum_{n=1}^{N1} \sum_{m=1}^{N1} Y_{ni} Y_{mj}^{*} \int_0^{\pi} \int_0^{2\pi} [E_n^{\theta} E_m^{*\theta} + E_n^{\phi} E_m^{*\phi}] \sin\theta \, d\phi d\theta \right] . \tag{15}$$

We suppose that

$$S_{mn} = \int_0^{\pi}\int_0^{2\pi} [E_n^{\theta}E_m^{*\theta} + E_n^{\phi}E_m^{*\phi}]\sin\theta \, d\phi d\theta \tag{16}$$

and by Equation (16)

$$\langle\vec{\phi}_i,\vec{\phi}_j\rangle = \left[\sum_{n=1}^{N1}\sum_{m=1}^{N1} Y_{ni}Y_{mj}^{*} \, S_{mn}\right] = K_{ij} \quad . \tag{17}$$

We now orthonormalize the vectors $\vec{\phi}_1,\vec{\phi}_2,\cdots,\vec{\phi}_N$ with the aid of:

$$\vec{\psi}_1 = \frac{\vec{\phi}_1}{|\langle\vec{\phi}_1,\vec{\phi}_1\rangle|^{1/2}} \tag{18}$$

$$\vec{\psi}_n = \frac{\vec{\phi}_n - \sum_{j=1}^{n-1} \langle\vec{\phi}_n,\vec{\psi}_j\rangle\vec{\psi}_j}{|\langle A_n,A_n\rangle|^{1/2}} \tag{19}$$

where A_n is the numerator of Equation (19) and $\vec{\psi}_n$ is the corresponding orthonormalized function [22] expressed by the equation:

$$\vec{\psi}_n = \sum_{i=1}^{n} C_i^{(n)} \, \vec{\phi}_i \quad . \tag{20}$$

By Equations (19) and (20) we can find the coefficients $C_i^{(n)}$ which will be

$$C_n^{(n)} = \frac{1}{|\langle\vec{\psi}_n,\vec{\psi}_n\rangle|^{1/2}} \tag{21}$$

$$C_k^{(n)} = - \frac{\sum_{j=k}^{n-1} C_k^{(j)} \left[\sum_{i=1}^{j} C_i^{(j)} \langle\vec{\phi}_i,\vec{\phi}_j\rangle\right]}{|\langle\vec{\psi}_n,\vec{\psi}_n\rangle|^{1/2}} \quad . \tag{22}$$

In view of Equation (17) the factors C take on the form

$$\left.\begin{aligned}
C_k^{(n)} &= -\frac{\sum_{j=k}^{n-1} C_k^{(j)} \left(\sum_{i=1}^{j} C_i^{(j)} K_{ni} \right)}{D_n} \\
C_n^{(n)} &= \frac{1}{D_n} \\
D_n &= \left\{ K_{nn} - \sum_{j=1}^{n-1} \left| \sum_{i=1}^{j} C_i^{(j)} K_{ni} \right|^2 \right\}^{1/2}
\end{aligned}\right\} \quad . \tag{23}$$

The expression of k_{ni} can be found as a function of the array geometry and is given in Appendix I. The electric field can now be expressed with the help of $\vec{\psi}_i$ as:

$$\vec{E}(\phi,\theta) = \sum_{i=1}^{N} L_i \, \vec{\psi}_i(\phi,\theta) \tag{24}$$

where the L_i are given by

$$L_i = \langle \vec{E}, \vec{\psi}_i \rangle \tag{25}$$

and the corresponding voltages V_i by the help of Equation (20) will be:

$$V_i = \sum_{j=i}^{N} L_j C_i^{(j)} \quad . \tag{26}$$

As result from the above discussion the synthesis of an array will follow these steps:

i) Definition of the five coordinates of each segment (three positional and two directional);

ii) Calculation of the constants C;

iii) Evaluation of the field $\vec{E}(\phi,\theta)$;

iv) Calculation of L_i; and

v) Computation of the feed voltages V_i.

IV. OPTIMIZATION PROBLEM

The optimization problem is a procedure of maximization of an index of the array with or without constraints. Some of the usual performance indices which are involved follow.

1. Power Gain

$$G = 4\pi \frac{\text{radiation intensity for specified direction}}{\text{power input to the array}} \quad . \tag{27}$$

In relation to the feed voltages and the input admittances, Equation (27) is expressed as

$$G(\phi_o,\theta_o) = \frac{\left| \sum_{i=1}^{N} V_i \, \vec{\phi}_i(\phi_o,\theta_o) \right|^2}{15 \sum_{i=1}^{N} \sum_{j=1}^{N} V_i V_j^* (Y_{ij}^r + Y_{ij}^{*r})} \tag{28}$$

where Y_{ij}^r is the admittance between the main ports i,j.

2. Directive Gain and Directivity

The directive gain is defined by

$$D = 4\pi \frac{\text{radiation intensity for specified direction}}{\text{radiation power}} \quad . \tag{29}$$

It can be shown that the radiation power is an expression of the form:

$$P_r = \sum_{i=1}^{N} \sum_{j=1}^{N} V_i V_j^* (Y_{ij}^r + Y_{ij}^{*r} - \overline{Y}_{ij} - \overline{Y}_{ij}^*) \tag{30}$$

where complete details for the $\overline{Y}_{ij}$ are available in Reference [28].

The expression of directive gain becomes

$$D(\phi_o,\theta_o) = \frac{\left| \sum_{i=1}^{N} V_i \, \vec{\phi}_i(\phi_o,\theta_o) \right|^2}{P_r} \tag{31}$$

and the maximum value of $D(\phi_o \theta_o)$ is the directivity.

3. Efficiency Indices

The efficiency indices are very important parameters describing the performance of an array. Many indices can be defined by several ways. Two of these are

$$S = \frac{\text{radiation intensity in the direction of max. radiation}}{\text{sum of the excitation voltages magnitudes squared}} \tag{32}$$

$$S_1 = \frac{\text{radiation intensity in the direction of max. radiation}}{\text{sum of the feed port current magnitudes squared}} . \tag{33}$$

Equations (32) and (33) can be written

$$S = \frac{\left| \sum_{i=1}^{N} V_i \vec{\phi}_i \right|^2}{\sum_{i-1}^{N} V_i V_i^{*}} \tag{34}$$

$$S_1 = \frac{\left| \sum_{i=1}^{N} V_i \vec{\phi}_i \right|^2}{\sum_{i=1}^{N} \left| \sum_{j=1}^{N} V_i Y_{ij}^{r} \right|^2} . \tag{35}$$

Some other factors as sensitivity and Q-factors can be defined. These are related to the above indices and given by

$$K = \frac{1}{S} \quad \text{and} \quad K_1 = \frac{1}{S_1} \quad \text{(sensitivity factors)} \tag{36}$$

$$Q = \frac{G}{S} \quad \text{and} \quad Q_1 = \frac{G}{S_1} \quad \text{(Q-factors).} \tag{37}$$

A. <u>Gain-Directive Gain and Efficiency Indices Maximization</u>

Until now we have seen that all the above indices can be written in the following formula

$$I_n(\phi_o,\theta_o) = \frac{\left| \sum_{i=1}^{N} V_i \; \vec{\phi}_i(\phi_o,\theta_o) \right|^2}{\sum_{i=1}^{N} \sum_{j=1}^{N} V_i V_j^* S_{ij}} \tag{38}$$

where S_{ij} is expressed correspondingly in Equations (28), (31), (34) and (35).

By the orthogonal method in the denominator of Equation (38) we can find an expression

$$\sum_{i=1}^{N} \sum_{j=1}^{N} V_i V_j^* S_{ij} = \sum_{i=1}^{N} L_i L_i^* \quad . \tag{39}$$

The L_i are related with V_i by Equation (26). The necessary factors $C_i^{(j)}$ are given from Equation (23) by substituting the K_{ij} by S_{ij}. Using the expression

$$\vec{\psi}_j(\phi_o,\theta_o) = \sum_{i=1}^{j} C_i^{(j)} \; \vec{\phi}_i(\phi_o,\theta_o) \tag{40}$$

Equation (38) yields

$$I_n(\phi_o,\theta_o) = \frac{\left| \sum_{i=1}^{N} L_i \; \vec{\psi}_i(\phi_o,\theta_o) \right|^2}{\sum_{i=1}^{N} L_i L_i^*} \quad . \tag{41}$$

we have

$$I_n(\phi_o,\theta_o) = \frac{\left|\sum_{i=1}^{N} L_i\, \psi_i^{\theta}\right|^2 + \left|\sum_{i=1}^{N} L_i\, \psi_i^{\phi}\right|^2}{\sum_{i=1}^{N} L_i L_i^*} \tag{42}$$

where $\psi_i^{\theta} = \vec{\psi}_i \cdot \hat{\theta}$ and $\psi_i^{\phi} = \vec{\psi}_i \cdot \hat{\phi}$. By the Schwartz inequality we can take

$$I_n^{\theta}(\phi_o,\theta_o) = \frac{\left|\sum_{i=1}^{N} L_i\, \psi_i^{\theta}\right|^2}{\sum_{i=1}^{N} L_i L_i^*} \le \frac{\left(\sum_{i=1}^{N} L_i L_i^*\right)\left(\sum_{i=1}^{N} \psi_i^{\theta}\psi_i^{*\theta}\right)^2}{\sum_{i=1}^{N} L_i L_i^*} = \sum_{i=1}^{N} \psi_i^{\theta}\psi_i^{*\theta} \tag{43}$$

and

$$I_n^{\phi}(\phi_o,\theta_o) = \frac{\left|\sum_{i=1}^{N} L_i\, \psi_i^{\phi}\right|^2}{\sum_{i=1}^{N} L_i L_i^*} \le \frac{\left(\sum_{i=1}^{N} L_i L_i^*\right)\left(\sum_{i=1}^{N} \psi_i^{\phi}\psi_i^{*\phi}\right)^2}{\sum_{i=1}^{N} L_i L_i^*} = \sum_{i=1}^{N} \psi_i^{\phi}\psi_i^{*\phi} \,. \tag{44}$$

From Equations (43) and (44) we can see that

$$I_n^{max} = \sum_{i=1}^{N} (\psi_i^{\theta}\psi_i^{*\theta} + \psi_i^{\phi}\psi_i^{*\phi}) \quad . \tag{45}$$

By the same procedure as in Reference [22] we can show that

$$L_i = k\, \psi_i^{*\theta} + \Lambda\, \psi_i^{*\phi} \tag{46}$$

where the parameters k and Λ are related by

$$\frac{1-K^2}{\Lambda^2-1} = \frac{\sum_{i=1}^{N} \psi_i^{\phi}\, \psi_i^{*\phi}}{\sum_{i=1}^{N} \psi_i^{\theta}\, \psi_i^{*\theta}} \qquad (47)$$

Equation (47) gives at the same time through K and Λ the polarization sought.

B. <u>Optimization Subject to Constraint on Electric Field Nulls and Sidelobe Levels</u>

Suppose that we want to determine the feed voltages that will provide electric field nulls and sidelobe levels in K directions for the E^{θ} component and in λ directions for the E^{ϕ} component. The above constraint give K+λ relations between the feed voltages. If the field in the K direction is related with the E_0^{θ} in the direction of maximum by the form:

$$\begin{bmatrix} E_1^{\theta} \\ E_2^{\theta} \\ \cdot \\ \cdot \\ \cdot \\ E_K^{\theta} \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ \cdot \\ \cdot \\ \cdot \\ K_K \end{bmatrix} E_0^{\theta} = [K]E_0^{\theta} \qquad (48)$$

and correspondingly for the E_0^{ϕ} field·

$$\begin{bmatrix} E_1^{\phi} \\ E_2^{\phi} \\ \cdot \\ \cdot \\ \cdot \\ E_{\lambda}^{\phi} \end{bmatrix} = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \Lambda_{\lambda} \end{bmatrix} E_0^{\phi} = [\Lambda]E_0^{\phi} \qquad (49)$$

then by the help of the Equation 10) we can find a relation between the $K+\lambda$ feed voltages and the rest $N-K-\lambda$. More details for this relation can be found in Reference [20]. Finally we will have a modified expression of $I_n(\phi_o,\theta_o)$ which will be of the form

$$I_n(\phi_o,\theta_o) = \frac{\left| \sum_{i=1}^{N-K-\lambda} V_i \vec{Z}_i(\phi_o,\theta_o) \right|^2}{\sum_{i=1}^{N-K-\lambda} \sum_{j=1}^{N-K-\lambda} V_i V_j^* g_{ij}} \quad . \tag{50}$$

Equation (50) is similar to Equation (38) and the maximization of the index I_n will be done by the same method as before.

The g_{ij} are the matrix elements [Q] of Equation (58) and the $\vec{Z}_i$ are the elements of the vector

$$[\vec{Z}] = [\Sigma_1]\hat{\theta} + [\Sigma_2]\hat{\phi}$$

of Equation (56) in Reference [20].

The above procedure gives the maximum index I_n when $K+\lambda$ values of the electric field are known.

If we want some of these to be the sidelobe levels then we can use an iterative procedure. Thus we find first the feed voltages by the above procedure and the level of the electric field sidelobes. These are compared with the desired levels. The new directions of sidelobes are used to find new feed voltage, etc. The procedure is continued until all the sidelobes take on the desired values.

V. EXAMPLES

A. Synthesis

In all examples we assume that the wires have radius 0.005λ, are centerfed and their length is $\lambda/2$. For the case of synthesis a double integration is applied. In Appendix II we show a method of numerical integration with good accuracy. This method is extended in Reference [30] and uses the Chebyshev polynomials.

1. Let us suppose at first that we have two dipoles normal to each other with their centers a distance about equal to the diameter of the wire. We wish to have an electric field of the form

$$\vec{E}(\phi,\theta) = e^{j\phi}\,\hat{\phi}$$

By the orthogonal method we found the feed voltages

$$V_1 = 1. \quad \text{and} \quad V_2 = -j \qquad .$$

The above values give the well known omnidirectional dipole.

2. Suppose a directive field of the form:

$$\vec{E}(\phi,\theta) = \exp(-a_1\sin^2\theta\ \cos^2\phi)\ \cos(2b_1\sin\theta\ \cos\phi)\hat{\theta} +$$

$$+ \exp(-a_2\sin^2\theta\ \cos^2\phi)\ \cos(2b_2\sin\theta\ \cos\phi)\hat{\phi}$$

has the following properties:

The E^{θ} is max. when $\sin\theta\cos\phi=0$, zeroing of the E^{θ} mainlobe is when $\sin\theta\cos\phi=\pi/4b_1$ and the beamwidth of E^{θ} is $2\sin\theta_1\cos\phi_1$ when $\sqrt{2}/2=\exp(-a_1^2 \sin^2\theta_1\cos\phi_1)\cos(2b_1\sin\theta_1\cos\phi_1)$. In the same way we can find the E^{ϕ}. An array with $N=7$ dipoles in equal distance $\lambda/2$ and with directions $\phi^{i}=0$, $\theta^{i}=\pi/4$ as shown in Figure 2, for $a_1=a_2=2.683$ and $b_1=b_2=1.428$ gives the following feed voltages $V_1=4.237\underline{/0°}$, $V_2=V_3=3.735\underline{/5°}$, $V_4=V_5=1.76\underline{/8°.3}$ and $V_6=V_7=.27\underline{/8°.1}$.

If Figures 3a,b we can see the normalized E^{θ} and E^{ϕ} field as a function of $p=\sin\theta$ for $\phi=0°$.

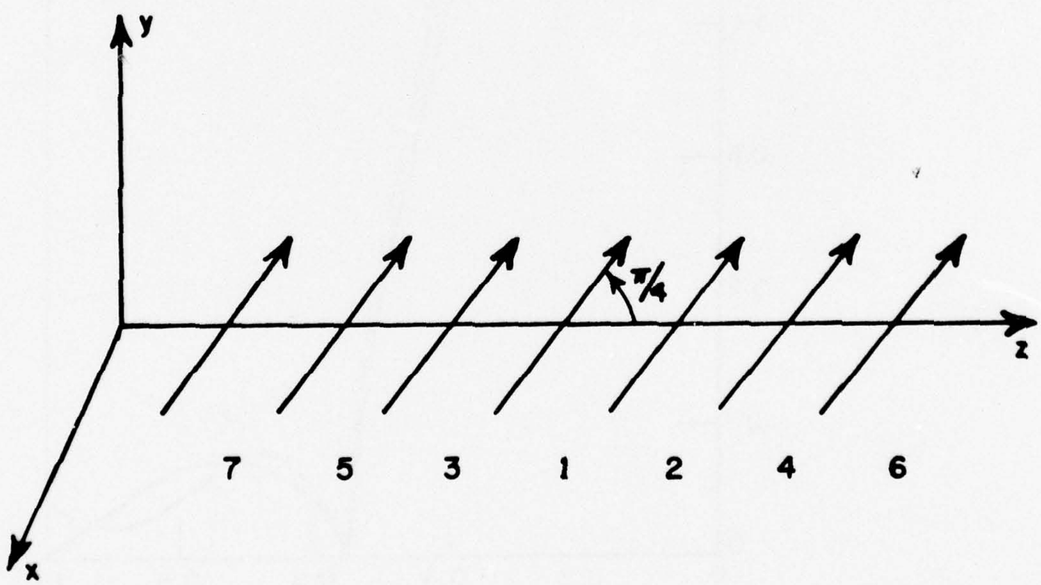


Figure 2. Seven dipole linear array of equal $\lambda/2$ spacing having orientation $\phi^{i}=0,\theta^{i}=\pi/4$.

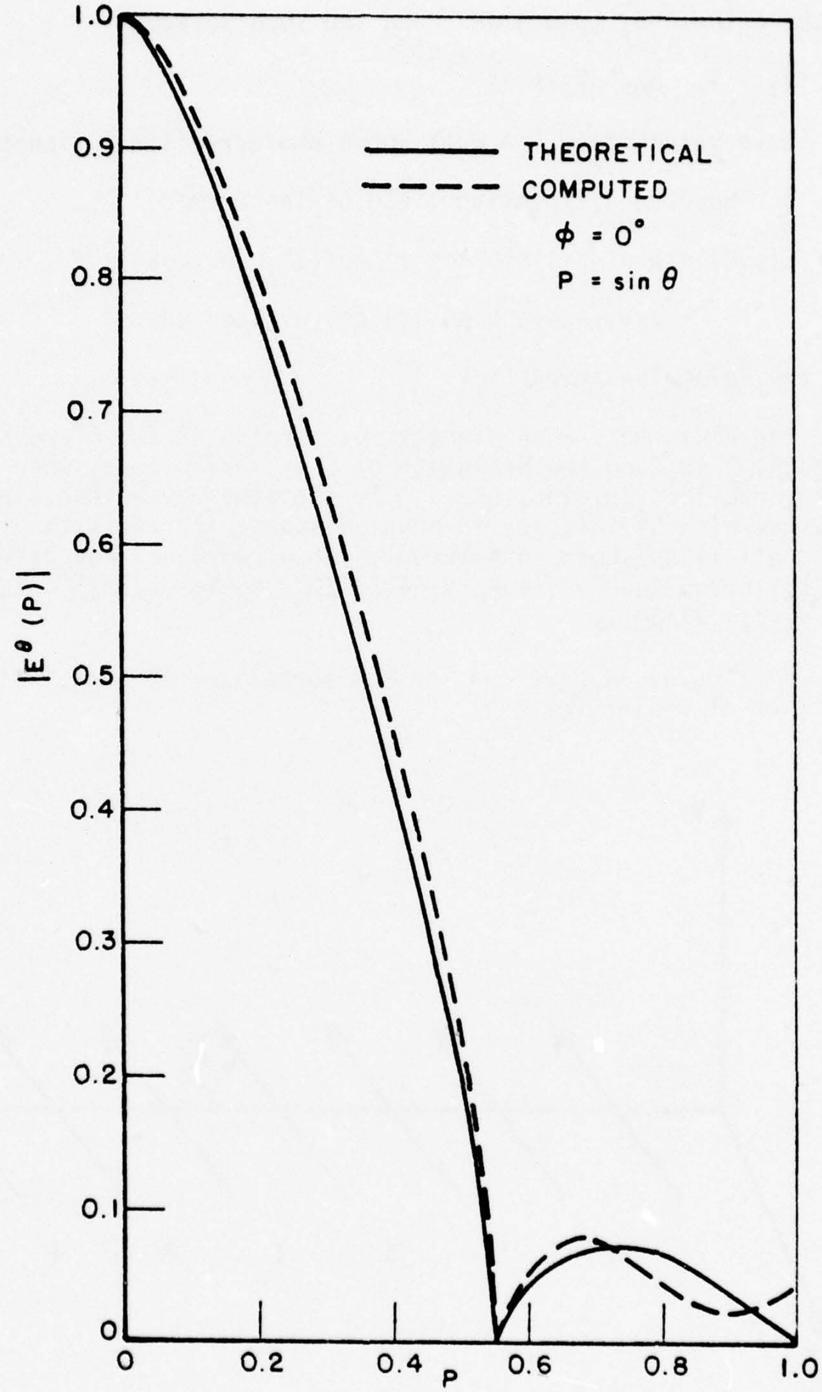


Figure 3a. Normalized E^θ field as a function of $p=\sin\theta$.

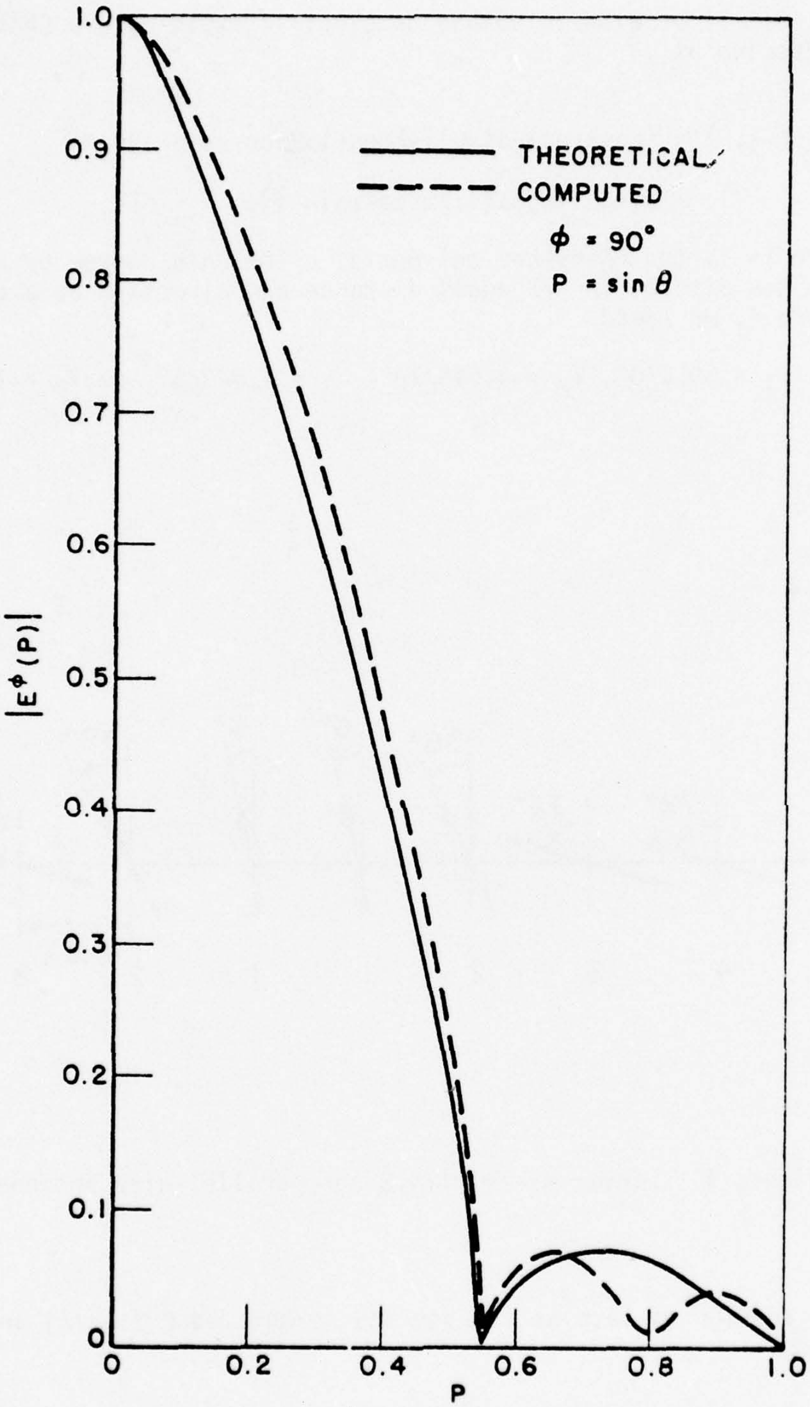


Figure 3b. Normalized E^{ϕ} field as a function of $p=\sin\theta$.

3. If we wish to obtain an electric field with a Chebyshev expression as

$$\vec{E}(\phi,\theta) = (\cos\theta\cos\phi - \sin\theta)T_7[\cos(1.2\pi\cos\phi\sin\theta)]\hat{\theta} +$$

$$- \sin\phi \; T_7[\cos(1.15\pi\cos\phi\sin\theta)]\hat{\phi}$$

where T_7 is the Chebyshev polynomial of seventh degree by using an array with N=8 dipoles in $\lambda/2$ equal distance and direction as are shown in Figure 4, we found

$$V_1 = 30.6\underline{/0^\circ},\ V_2 = 4.645\underline{/10^\circ},\ V_3 = 1.843\underline{/5^\circ} \text{ and } V_4 = 1.0\underline{/0^\circ}.$$

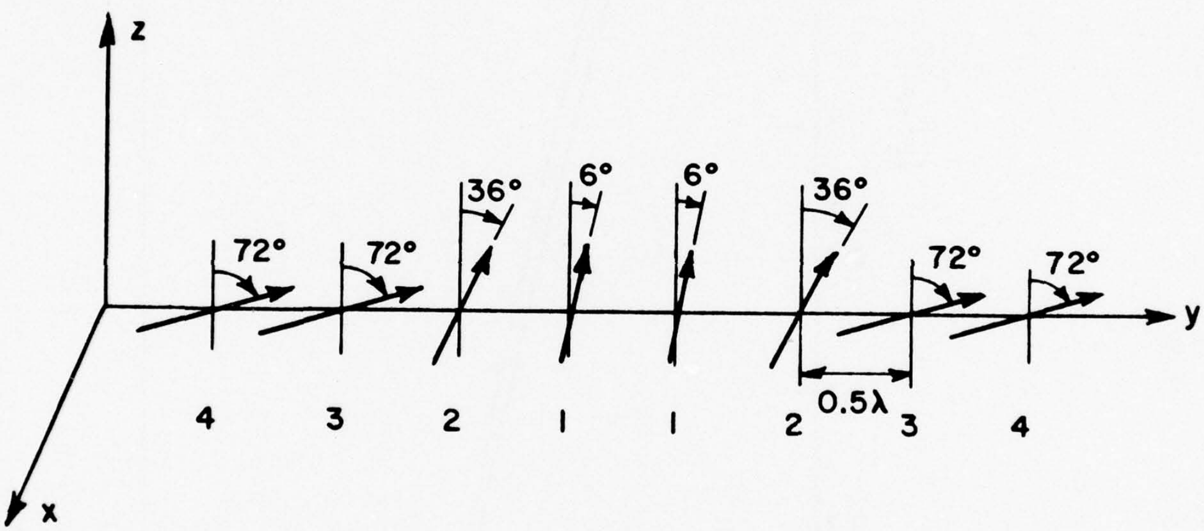


Figure 4. Linear array with 8 non-parallel wire antennas.

In Figures 5a,b we can see the normalized $E^{\theta}(\phi,\pi/2)$ and $E^{\phi}(\phi,\pi/2)$ fields.

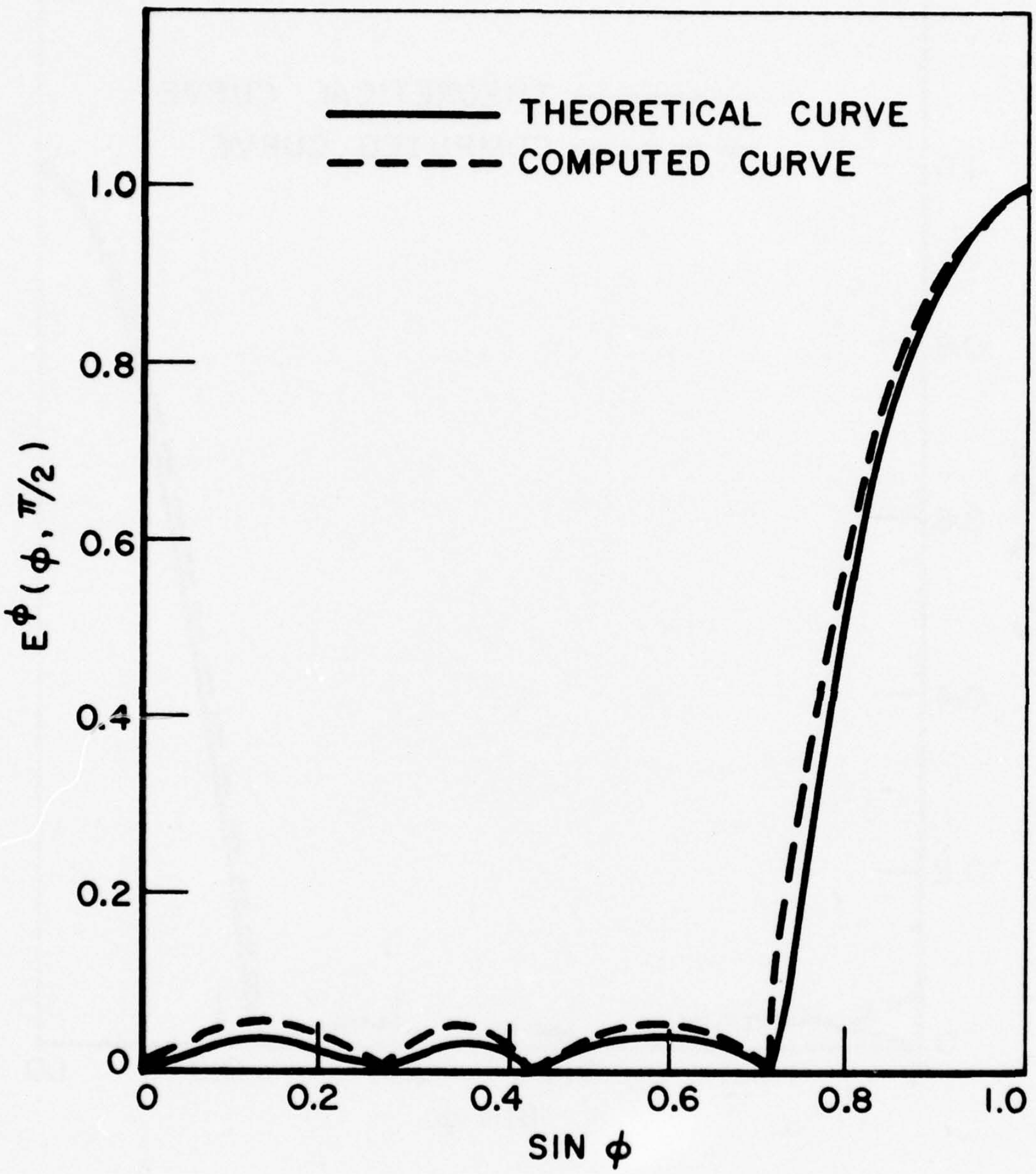


Figure 5a. Normalized $E^{\theta}(\phi,\pi/2)$ field of the linear array in Figure 4.

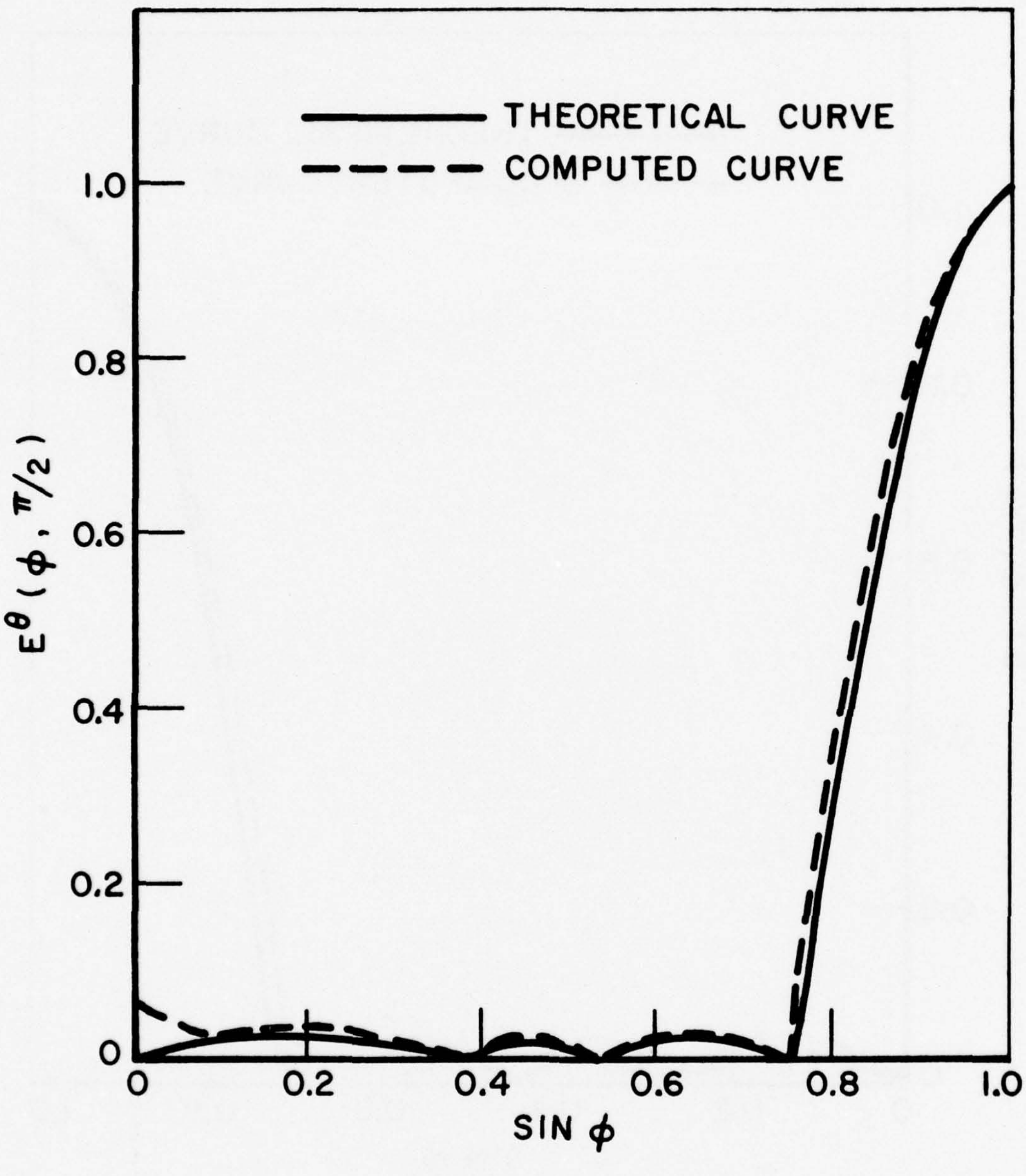


Figure 5b. Normalized $E^{\phi}(\phi,\pi/2)$ field of the linear array in Figure 4.

B. Optimization

In Figure 6 we can see the maximum gain of a linear uniform array of nonparallel wire antennas with $\phi^i=0$ and $\theta^i=2\pi(i-1)/N$ for N=4 to 10 versus the interelement distance in the broadside direction. It is interesting to say that in this case there are critical equaldistances in which we have maxima. So in .7λ equal distance we have the maximum gain for all arrays.

In Figure 7 is plotted the maximum directive gain of a circular array with tangential wire antennas in the directions $\theta^i=\pi/4$, $\phi^i=2\pi(i-1)/N+\pi/2$ for N=4 to 10 versus the diameter of the array. In this case the directive gain increases as the diameter decreases.

Another one example is the design of a 6-element linear array in equal distance .5λ and directions $\phi^i=0$, $\theta^i=\pi(i-1)/10$ to provide maximum gain in the broadside direction subject to the contraint that for both E^θ and E^ϕ are required nulls in the direction ($\phi=50°,\theta=90°$). We found maximum gain G=11.559 and the feed voltages

$$V_1 = .088\underline{/-124°.6},\quad V_2 = .246\underline{/-175°.5},\quad V_3 = .6576\underline{/-250°}$$

$$V_4 = 1\underline{/0°},\quad V_5 = .285\underline{/40°},\quad V_6 = .4259\underline{/-34°.6} \quad .$$

In Figure 8 we can see the normalized $E^\theta(\phi,\pi/2)$ and $E^\phi(\phi,\pi/2)$ fields.

As a last example the same 6-element array has maximum gain G=6.767 subject to the constraints that (i) nulls are required to direction ($\phi=60°,\theta=90°$) and ($\phi=50°,\theta=90°$) for the E^θ and E^ϕ fields correspondingly and (ii) $E^\theta(40°,90°)=1/5\ E^\theta(90°,90°)$. In Figure 9 we can see the normalized $E^\theta(\phi,\pi/2)$ and $E^\phi(\phi,\pi/2)$ fields. The corresponding feed voltages in this case in absolute values as they were taken from the computer output are:

$$V_1 = 57.578 + j38.129,\quad V_2 = 81.02 + j29.205,\quad V_3 = 91.412 + j52.613$$

$$V_4 = 100.834 + j30.476,\quad V_5 = 61.564 + j6.619,\quad V_6 = 44.851 + j6.3057 \quad .$$

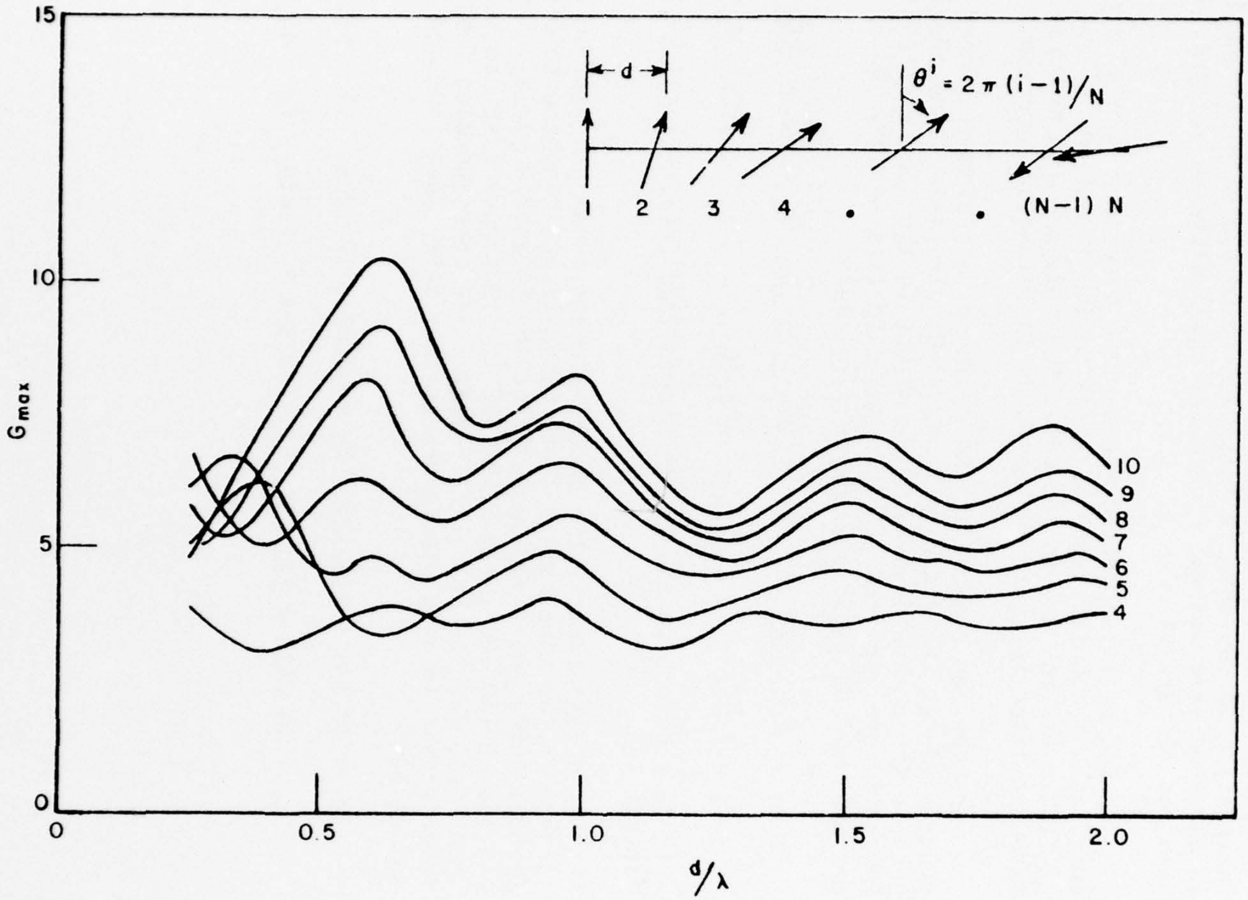


Figure 6. Maximum gain of nonparallel wire antennas with $\phi^i=0°, \theta^i=2\pi(i-1)/N$ for N=4 to 10 elements versus the interelement distance.

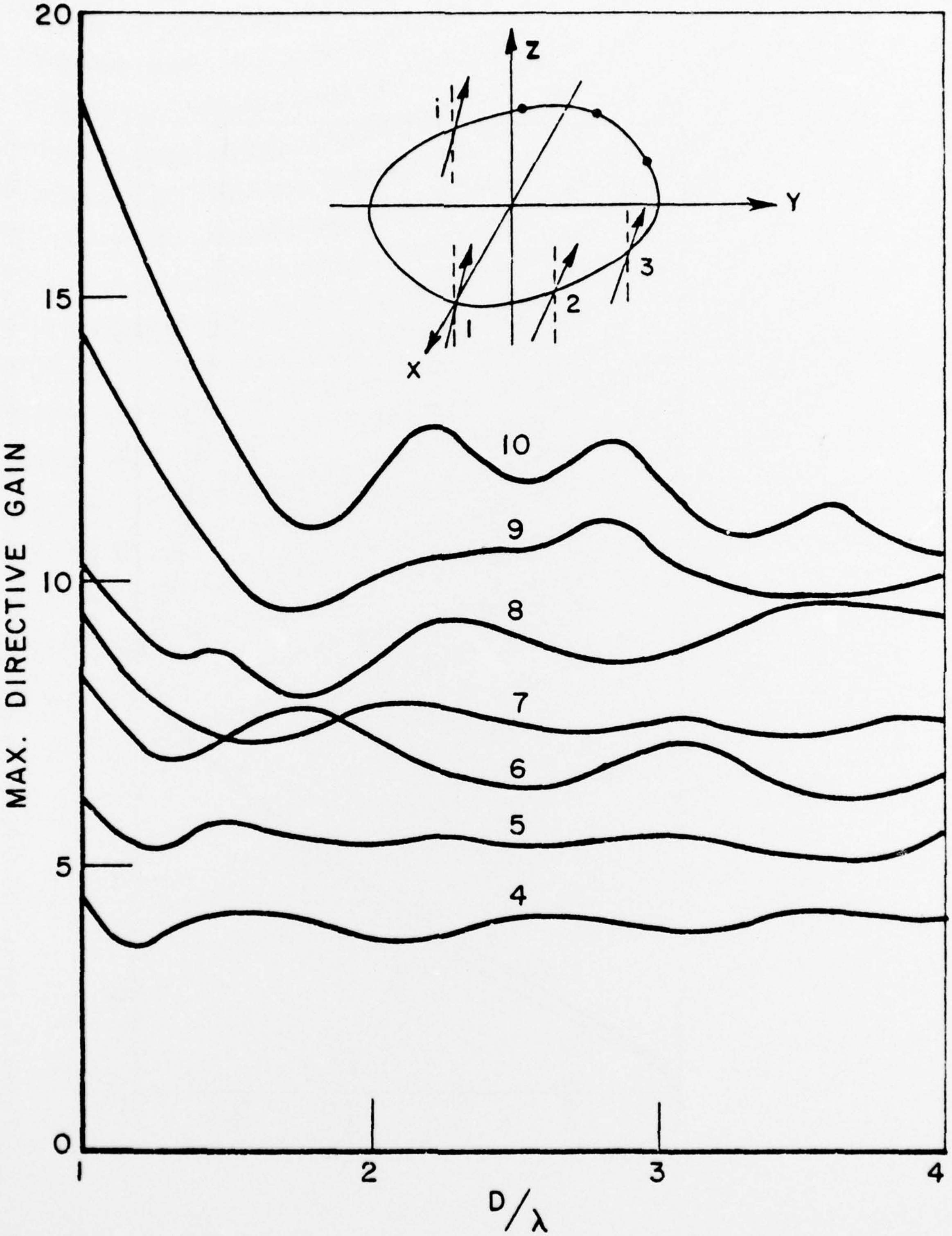


Figure 7. Maximum directive gain of a circular array with tangential wire antennas in the direction $\theta^i=\pi/4, \phi^i=\pi/2+2(i-1)/N$ for $N=4$ to 10 elements versus the diameter of the array.

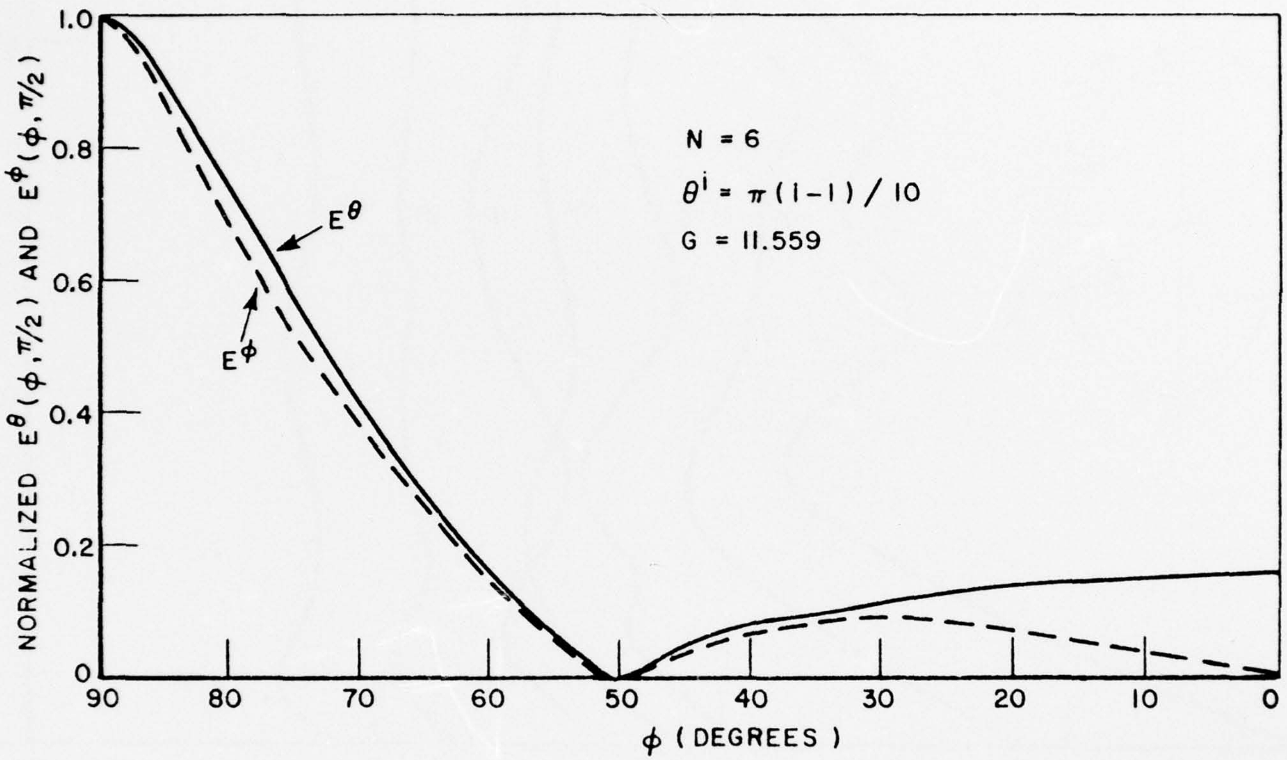


Figure 8. Normalized $E^{\theta}(\phi,\pi/2)$ and $E^{\phi}(\phi,\pi/2)$ fields of a linear array with maximum gain in the broadside direction and a null at ϕ=50°,θ=90°.

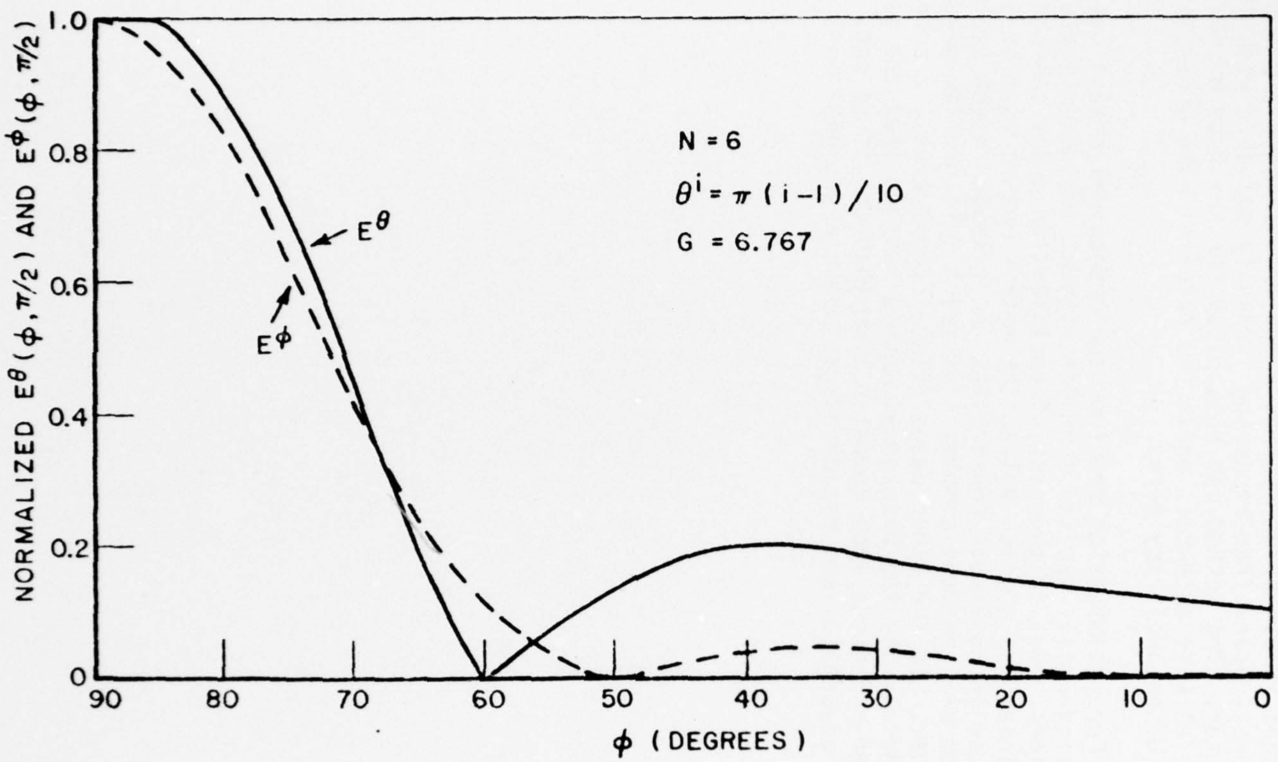


Figure 9. Normalized $E^{\theta}(\phi,\pi/2)$ and $E^{\phi}(\phi,\pi/2)$ fields of a linear array with maximum gain and: (i) null in direction (ϕ=60°, θ=90°) for the E^{θ} and (ϕ=50°, θ=90°) for the E^{ϕ} field, (ii) $E^{\theta}(40°,90°)=1/5\ E^{\theta}(90°,90°)$.

VI. CONCLUSION

As a result of the foregoing discussion, we have seen that the orthogonal method can be applied to an arbitrary array of wire antennas, and that it can solve synthesis and optimization problems with good accuracy, apart from rendering it easily solvable with the aid of a computer. The method can be applied for any case of parallel or non-parallel wire antennas, while the parallel wire array is a special case of an arbitrary array.

For optimization problems the orthogonal method has the advantage of using easily applied formulas without the necessity of inverting matrices. But in the case of optimization under the constraint that one index has a given value, the matrix method must be used. That is because in nonlinear constraints the formulas given by the orthogonal method will be more complex. At least this problem needs to be studied further. In the other cases (linear constraints) a comparison between the Matrix and the Orthogonal method reveals that the second one is faster and needs less computer time than the first one. So, both methods have advantages and disadvantages which a designer must keep in mind.

APPENDIX I

From Equation (17) we have found that

$$K_{ij} = \left[\sum_{n=1}^{N1} \sum_{m=1}^{N1} Y_{ni} Y^*_{mj} S_{mn} \right] \tag{I-1}$$

From the expression (16) we can find that S_{mn} as a function of the array geometry, so

$$S_{mn} = \int_0^{\pi} \int_0^{2\pi} [E_{n\theta} E^*_{m\theta} + E_{n\phi} E^*_{m\phi}] \sin\theta \, d\theta d\phi \quad . \tag{I-2}$$

By using Equations (11) and (12) and integrating we take

$$S_{nm} = \sin\theta^m \sin\theta^n (A_{mn} + B_{mn}) + \sin\theta^m \cos\theta^n \, C^m_{mn} +$$

$$+ \sin\theta^n \cos\theta^m \, C^n_{mn} + \cos\theta^m \cos\theta^n \, G_{mn} \tag{I-3}$$

where

$$A_{mn} = 2\pi \cos[2(\phi_m - \phi_n) + \phi^m + \phi^n] \sqrt{\frac{\pi}{2}} \, (KR_{mn})^2 \; \frac{J_{5/2}(Kr_{mn})}{(Kr_{mn})^{5/2}} \tag{I-4}$$

$$B_{mn} = 2\pi \cos(\phi^m - \phi^n) \quad 2\sqrt{\frac{\pi}{2}} \, \frac{J_{1/2}(Kr_{mn})}{(Kr_{mn})^{1/2}} - 2\sqrt{\frac{\pi}{2}} \, \frac{J_{3/2}(Kr_{mn})}{(Kr_{mn})^{3/2}} \; +$$

$$+ \; \frac{\pi}{2} \; (KR_{mn})^2 \, \frac{J_{5/2}(Kr_{mn})}{(Kr_{mn})^{5/2}} \quad . \tag{I-5}$$

$$C^m_{mn} = 4\pi \cos(\phi_m - \phi_n - \phi^m) \sqrt{\frac{\pi}{2}} \, (K^2 R_{mn} Z_{mn}) \; \frac{J_{5/2}(Kr_{mn})}{(Kr_{mn})^{5/2}} \tag{I-6}$$

$$G_{mn} = 4\pi \sqrt{\frac{\pi}{2}} \left\{ 2 \frac{J_{3/2}(Kr_{mn})}{(Kr_{mn})^{3/2}} - (KR_{mn})^2 \frac{J_{5/2}(Kr_{mn})}{(Kr_{mn})^{5/2}} \right. \qquad \text{(I-7)}$$

when $m=n$

$$S_{mn} = \frac{8\pi}{3} \quad .$$

By knowing the S_{mn} the K_{ij} is known from Equation (I-1).

In Equations (4) - (7) the involving parameters are:

$$k = \frac{2\pi}{\lambda} \quad ,$$

$$r_{mn} = \sqrt{(X_m-X_n)^2+(Y_m-Y_r)^2+(Z_m-Z_r)^2} \quad ,$$

$$R_{mn} = \sqrt{(X_m-X_n)^2+ (Y_m-Y_n)^2} \quad ,$$

$$Z_{mn} = (Z_m-Z_n) \quad .$$

APPENDIX II

INTEGRATIONS INVOLVED ON SYNTHESIS

For the case of the synthesis as we can see from Equation (25) one double integration is applied. The electric field which is desired may or may not be one function with a closed form mathematical expression.

In all cases the description of the electric field may be done by sample points which we will show as follows.

The inner product of Equation (25) is an expression of the form

$$L_i = \int_0^{\pi} \int_0^{2\pi} [E^{\theta}_{(\phi,\theta)} \psi_i^{\theta *}(\phi,\theta) + E^{\phi}(\phi,\theta)\psi_i^{\phi *}(\phi,\theta)] \sin\theta \, d\phi d\theta \qquad \text{(II-1)}$$

A suitable formula for this double integration, as well as for a single one, was found by Sahalos [29-30] by the help of Chebyshev polynomials.

If we have the integration

$$J = \int_a^b \int_c^d f(\phi,\theta) d\phi d\theta \qquad \text{(II-2)}$$

we can modify the limits of integration as

$$\phi = \frac{1}{2}[(a+b) + (b-a)q]$$
$$\theta = \frac{1}{2}[(d+c) + (d-c)p] \qquad \text{(II-3)}$$

and have

$$J = \frac{(b-a)(d-c)}{4} \int_{-1}^{1} \int_{-1}^{1} f(p,q) \cdot dp \cdot dq \quad . \qquad \text{(II-4)}$$

With n^2 sample points the Chebyshev polynomials[30] gives an approximation to Equation (II-4) of the form

$$J = \frac{(b-a)(d-c)}{4} \cdot \frac{16}{n^2} \sum_{k=0}^{n/2}{}'' \sum_{s=0}^{n} \sum_{\ell=0}^{n/2}{}'' \sum_{m=0}^{n}{}'' \frac{\cos\frac{2k\pi s}{n} \cdot \cos\frac{2m\pi\ell}{n}}{(4k^2-1)(4\ell^2-1)}$$

$$\cdot\ f\left(\cos\frac{\pi s}{n}, \cos\frac{\pi\ell}{n}\right) \quad . \tag{II-5}$$

(Σ'' means that the first and last coefficients of the sum are multiplied by 1/2).

Expression (II-5) is written as

$$J \simeq \frac{4}{n^2}(b-a)(b-c) \sum_{s=0}^{n}{}'' \sum_{\ell=0}^{n}{}'' A(s,\ell)\cdot f\left(\cos\frac{\pi s}{n}, \cos\frac{\pi\ell}{n}\right) \tag{II-6}$$

the $A(s,\ell)$ is the weight of the point with $q = \cos(\pi s/n)$ and $p = \cos(\pi\ell/n)$.

By using 8^2 points for example we can make a table with $A(s,\ell)$ which by putting to the memory of the computer can have more faster the integration. In the following table we can see the $A(s,\ell)$ for N=8.

Some examples to show the accuracy of the method follow.

1. The integral

$$J = \int_4^{5.2}\int_2^{3.2} \frac{dydx}{xy}$$

has an accurate value equal to 0.123312. Application of the formula (55) gave the following table:

n	J
6	0.123 312 156 4
8	0.123 312 156 484
10	0.123 312 156 484 11

as we can see for n=8 we have an approximation with 12 significant figures.

N=8

L→ S ↓	0	1	2	3	4	5	6	7	8
0	.30864	.44465	.58377	.68410	.71076	.68410	.58377	.44465	.30864
1	.44465	.6406	.84103	.98556	1.02397	.98556	.84103	.6406	.44465
2	.58377	.84103	1.10417	1.29392	1.34435	1.29392	1.10417	.84103	.58317
3	.68410	.98556	1.29392	1.51628	1.57537	1.51628	1.29392	.98556	.68410
4	.71076	1.02397	1.34435	1.57537	1.63678	1.57537	1.34435	1.02392	.71076
5	.68410	.98556	1.29392	1.51628	1.57537	1.51628	1.29392	.98556	.68410
6	.58377	.84103	1.10417	1.29392	1.34435	1.29392	1.10417	.84103	.58377
7	.44465	.6406	.84103	.98556	1.02397	.98556	.84103	.6406	.44465
8	.30864	.44465	.58377	.68410	.71076	.68410	.58377	.44465	.30864

2. The integral

$$J = \int_0^{\pi} \int_0^{2\pi} \exp j2\pi\left(\frac{1}{3}\sin\theta\cos\phi + \frac{1}{2}\sin\theta\sin\phi + \frac{1}{6}\cos\theta\right) \sin\theta \, d\theta d\phi$$

has an accurate value of -2.247888336+j0. Application of the formula (II-5) gives

n	J
6	-2.2478 +j 0.
8	-2.247 888 +j 0.
10	-2.247 888 336 1 +j0.

3. By using Simpson's rule the integral

$$J = \int_1^3 \int_1^3 \sin^2 x \sin^2 y \, dxdy$$

gives seven significant figures for n=32. The exact value of the integral is 1.68267136378. Our formula gives:

n	J
6	1.682 681
8	1.682 671 36
10	1.682 671 363 781

Thus convinced that the approximate integration of the function $f(\phi,\theta)$ presents a high degree of accuracy, we went on to the synthesis of the antenna.

REFERENCES

1. C.L. Dolph, "A Current Distribution for Broadside Arrays Which Optimizes the Relationship Between Beamwidth and Sidelobe Level," Proc. IRE, Vol. 34, June 1946, pp. 335-348.

2. H.J. Riblet, "Discussion on Current Distribution for Broadside Arrays Which Optimizes the Relationship Between Beamwidth and Sidelobe Level," Proc. IRE, Vol. 35, May 1947, pp. 489-492.

3. H. Unz, "Linear Arrays with Arbitrarily Distributed Elements," Electron. Res. Lab., University of California, Berkeley. Rep. Ser. 60, No. 168, November 2, 1956.

4. H. Unz, "Multi-dimensional Latice Arrays with Arbitrarily Distributed Elements," Electron. Res. Lab., University of California, Berkeley, Rep. Ser. 60, No. 172, December 19, 1956.

5. H. Unz, "New Methods for Synthesis of Nonuniformly Spaced Antenna Arrays," Report 1423-1, 15 November 1962, The Ohio State University Antenna Laboratory, Department of Electrical Engineering; prepared under Contract AF 39(602)-2711 for Rome Air Development Center. (AD 405866) (RADC-TDR-63-131)

6. H. Unz, "Linear Array With Arbitrarily Distributed Elements," IRE Trans. Antennas and Propagat., Vol. AP-8, March 1960, pp. 222-223.

7. R.F. Harrington, "Sidelobe Reduction of Nonuniform Element Spacing," IRE Trans. Antennas and Propagat., Vol. AP-9, March 1961, pp. 187-192.

8. D.D. King, R.F. Packard and R.K. Thomas, "Unequally Spaced, Broadband Antenna Arrays," IRE Trans. Antennas and Propagat., Vol. AP-8, July 1960, pp. 380-385.

9. H. Schjaer-Jacobsen and K. Madsen, "Synthesis of Nonuniformly Spaced Arrays Using a General Nonlinear Minimax Optimization Method," IEEE Trans. Antennas and Propagat., Vol. AP-24, July 1976.

10. A.I. Uzkov, "An Approach to the Problem of Optimum Directive Antenna Design," Compltes Rendus (Doklady) de ℓ' Academie de Sciences de ℓ' URSS, Vol. LIII, No. 1, 1946, p. 35.

11. A. Bloch, R.G. Medhurst and S.D. Pool, "A New Approach to the Design of Super-Directive Aerial Arrays," J.I.E.E., 100, Part III, 1953, p. 303.

12. M. Uzsoky and L. Solymar, "Theory of Superdirective Linear Arrays," Acta Phys., Vol. 6, 1956, pp. 185-206.

13. C.O. Stearns, "Computer Performance of Moderate Size, Supergain Antennas," Boulder Labs., Nat. Bur. Stand. U.S. Dept. Commerce, Boulder, Colorado, Rep. 6796, September 1961.

14. C.T. Tai, "The Optimum Directivity of Uniformly Spaced Broadside Arrays of Dipoles," IEEE Trans. Antennas and Propagat., Vol. AP-12, 1964, pp. 447-454.

15. D. Cheng, "Optimization Techniques for Antenna Arrays," Proc. IEEE, Vol. 54, 1971, pp. 1664-1676.

16. Y.T. Lo, S.W. Lee and Q.H. Lee, "Optimization of Directivity and SNR of an Arbitrary Antenna Array," Proc. IEEE, Vol. 54, 1971, pp. 1033-1045.

17. S.M. Sangire and J.K. Butler, "Constrained Optimization of the Performance Indices of Arbitrary Arrays Antennas," IEEE Trans., Antennas and Propagat., Vol. AP-19, 1971, pp. 493-498.

18. B.J. Strait and D.C. Kuo, "Optimization Methods for Arrays of Parallel Wire Antennas," Scient. Rep., Contract No. F19628-68-C-1080, Syracuse University, Syracuse, N.Y., 1972.

19. T.K. Sarkar and B.J. Strait, "Optimization Methods for Arbitrarily Oriented Arrays of Antennas in Any Environment," Radio Science, Vol. 11, No. 12, 1976, pp. 959-967.

20. J. Sahalos, "Optimization Methods for Arrays of Nonparallel Wire Antennas," Report 4372-1, March 1977, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract N00014-76-C-0573 for Department of the Navy, Arlington, Virginia.

21. J. Sahalos, "On the Design of Antennas Consisting of Arbitrarily Oriented Dipoles," IEEE Trans. Antennas and Propagat., Vol. AP-24, January 1976, pp. 106-109.

22. J. Sahalos, "On the Optimum Directivity of Antennas Consisting of Arbitrarily Oriented Dipoles," IEEE Trans. Antennas and Propagat., Vol. AP-24, May 1976, pp. 322-327.

23. H. Unz, "Nonuniformly Spaced Arrays: The Orthogonal Method," Proc. IEEE, Vol. 54, January 1966, pp. 53-54.

24. J. Sahalos, "The Orthogonal Method of Nonuniformly Spaced Arrays," Proc. IEEE, Vol. 62, February 1974, pp. 281-282.

25. J. Sahalos, "A Solution of the General Nonuniformly Spaced Antenna Array," Proc. IEEE, Vol. 62, No. 9, September 1974.

26. J. Sahalos, K. Melidis and S. Lanbou, "On the Optimum Directivity of General Nonuniformly Spaced Broadside Arrays of Dipoles," Proc. IEEE, Vol. 62, No. 12, December 1974.

27. J.H. Richmond, "Radiation and Scattering by Thin-Wire Structures in the Complex Frequency Domain," Report 2902-10, July 1973, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Grant NGL 36-008 138 for National Aeronautics and Space Administration.

28. E.H. Newman, J.H. Richmond and C.H. Walter, "Super-Directive Array Study," Report 3955-2, September 1975, The Ohio State University ElectroScience Laboratory, Department of Electrical Engineering; prepared under Contract DAAB03-74-C-0516(1433) for Department of the Army.

29. J. Sahalos, "A Solution of Non-Uniformly Linear Antenna Array With the Help of Chebyshev Polynomials," IEEE Trans. Antennas and Propagat., Vol. AP-24, March 1976, pp. 239-242.

30. J. Sahalos, "A Solution of Radiation Pattern Synthesis of Arbi Arbitrary Antennas," Sci. Annals. Phy. and Mathem., University of Thessaloniki, Vol. 16, 1976.